Algebra I

Chapter 31

I. Factorable Denominators

When we add rational expression we need to make sure we have a common denominator. If we have either a polynomial that can be factored or the difference of two squares in the denominator, it is easier to find a common denominator if we factor.

ex 1. $$\frac{x}{p^2-16}+\frac{p}{p^2-4p}$$

Factor the difference of squares in the first denominator and factor a *p* out of the second denominator. Here's what it will look like:

$$\frac{x}{(p-4)(p+4)}+\frac{p}{p(p-4)}$$

We can cancel a *p* in second term.

$$\frac{x}{(p-4)(p+4)}+\frac{1}{(p-4)}$$

We need to multiply the second term by $\frac{p+4}{p+4}$ to get a common denominator.

$$\frac{x}{(p-4)(p+4)}+\frac{p+4}{(p-4)(p+4)}$$

Now we can add the numerators over the common denominator.

$$\frac{x+p+4}{(p-4)(p+4)}$$

By factoring we have made it more obvious what the common denominator is.

Let's do another example.

ex 2. $$\frac{x-5}{x^2-7x+10}+\frac{-28}{x^3+10x^2-24x}$$

We can factor both denominators. Factor the first denominator into two binomials. Factor an *x* out of the second binomial and then factor the remaining trinomial.

$$\frac{x-5}{(x-2)(x-5)}+\frac{-28}{x(x^2+10x-24)}=\frac{x-5}{(x-2)(x-5)}+\frac{-28}{x(x-2)(x+12)}$$

In the first term we can cancel the (x – 5) 's. After we do this we need to multiply the first term by:

$$\frac{x(x+12)}{x(x+12)}$$

to get a common denominator:

$$\frac{x(x+12)}{x(x-2)(x+12)}+\frac{-28}{x(x-2)(x+12)}$$

Now we can add the numerators over the common denominator:

$$\frac{x(x+12)-28}{x(x-2)(x+12)}=\frac{x^2+12x-28}{x(x-2)(x+12)}$$

Factor the numerator and reduce:

$$\frac{(x+14)(x-2)}{x(x-2)(x+12)}=\frac{x+14}{x(x+12)}$$

II. Rational Equations

We've already worked some with rational equations, but now we're going to do a bit more with them. Remember that we solve an equation with rational expressions the same way we solve any equation: get the variable alone on one side of the equals sign.

ex 3. $\frac{x}{5}+\frac{2x+2}{5}=\frac{9}{5}$

First we need to combine the terms with *x*. This is easy because they both have the same denominator. We can just add the numerators over the common denominator.

$$\frac{x+2x+2}{5}=\frac{9}{5}$$

Now multiply both sides by 5 to get rid of the denominator.

$$x+2x+2=9$$

We can solve for *x* easily now. Combine like terms and –2 from both sides.

$$3x=7$$

$$x=\frac{7}{3}$$

Let's solve another rational equation.

ex 4. $\frac{8}{x}+\frac{2}{3}=\frac{4}{9}$

First move all the terms <u>without</u> *x* to one side of the equation.

$$\frac{8}{x}=\frac{4}{9}-\frac{2}{3}=\frac{4}{9}-\frac{6}{9}=-\frac{2}{9}$$

Multiply both sides by *x* to get it out of the denominator.

$$8 = -\frac{2}{9}x$$

$$-\frac{9}{2}(8) = x$$

x = -36

In this last example we multiplied by x to get it out of the denominator. We can do this with other expressions involving x also.

ex 5. $$\frac{5}{x} = \frac{8}{x+2}$$

If we multiply both sides by x and x + 2 we can get all of the x's out of the denominator right away.

$$5(x+2) = 8x$$

$$5x + 10 = 8x$$

$$10 = 3x$$

$$x = \frac{10}{3}$$

Let's do one more.

ex 6. $$\frac{5-x}{10x} + \frac{1}{10} = \frac{4x-5}{5x}$$

Move all the terms with an x to the left side and move all the terms without an x to the right side.

$$\frac{5-x}{10x} - \frac{4x-5}{5x} = -\frac{1}{10}$$

Put the two rational expressions over a common denominator and add them.

$$\frac{5-x}{10x}-\frac{8x-10}{10x}=-\frac{1}{10}$$

$$\frac{5-x-8x+10}{10x}=-\frac{1}{10}$$

$$\frac{15-9x}{10x}=-\frac{1}{10}$$

Multiply the entire equation by $10x$ to get x out of the denominator.

$$15-9x=-x$$

Now we again have x's on both sides so move them all to one side.

$$15=8x$$

$$x=\frac{15}{8}$$